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AN APPROACH TO RELIABILITY DETERMINATION OF A ROTATING COMPONENT SUBJECTED TO COMPLEX FATIGUE

by Vincent R. Lalli and Dr. Dimitri B. Kececioglu
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at
Ninth Reliability and Maintainability Conference cosponsored by the
American Institute of Aeronautics and Astronautics, the Society of
Automotive Engineers, and the American Society of Mechanical Engineers
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Vincent R. Lalli
Lewis Research Center
Cleveland, Ohio

and

Dr. Dimitri B. Kececioglu
University of Arizona
Tucson, Arizona

E-5528

ABSTRACT

A methodology for determining the reliability of mechanical components is given. The necessary design data are pointed out. Complex fatigue research machines are described, which are generating the required fatigue strength data in terms of cycles to failure and endurance strength. The test loading is that of alternating bending combined with steady torque. The design data obtained are presented. The design by reliability methodology and data are applied to the design of an actual alternator rotor inner shaft, and the results are compared with those obtained by conventional design procedure.

SYMBOLS

$\bar{A}$	average area, in.^{2}*
b	positive integer
c	specimen radius, in.
c'	power of ten exponent
D	specimen major diameter, in.
d	specimen groove diameter, in.
e	natural log base, 2.718 +
I	moment of inertia, in.^{4}
J	polar moment of inertia, in.^{4}
$K_{b,f,kw,s,t}$	stress concentration factor: b, bending; f, fatigue reduction; kw, keyway; s, torsional shear; t, part geometry
K_K	coefficient of kurtosis
K_L	life cycles factor
K_{SK}	coefficient of skewness
K_V	coefficient of variation

*Barred symbols are also used to represent statistically distributed variables

$k_{a,b,c,d,e,f}$	modifying factors
$L_{b,ut,y}$	tensile load (see S), klb
M	bending moment on specimen, in.-lb
N_f	cycles to failure
n	number of specimens
R	reliability
r	specimen groove radius, in.
r'	coefficient of correlation
r_s	stress ratio, s_a/s_m
$S_{a,m}$	graphical value material strength (see s), psi
$S_{b,ut,y}$	tensile strength, psi: b, breaking; ut, ultimate; y, yield
S_e	corrected endurance limit, psi
S_e'	estimated endurance limit of the rotating beam specimen, psi
$s_{a,m}$	material stress, kpsi: a, alternating; m, mean
T	torque on specimen, lb-in.
Tr	test results: s, success; f, failure
W_P	pan weight, lb
Z	standardized normal variable
Δ	tolerance, in.
θ	angle, deg
μ	normal mean estimate
ξ	difference statistic, psi
σ	normal standard deviation
σ_x	component of bending stress, psi: C, compresion; t, tension; a, alternating; m, mean
σ_y	component of radial stress, psi
τ	component of shear stress, psi: m, mean; a, alternating
∞	infinity

PRESENT MECHANICAL DESIGN techniques depend heavily on "modifying factors" to compensate for limited knowledge of materials and loads. Some of these "modifying factors" compensate for imperfect knowledge concerning such things as surface condition, size, temperature, stress concentration and miscellaneous effects. In the best cases they are based on empirical data obtained over a long period of time, but in many other cases there is little real data and the selection is based on the designer's feel at the time. A final factor is often applied which generally reflects all the uncertainties involved and is probably influenced as much by the seriousness of the consequences if the part should fail as by anything else. The resulting design is probably considerably heavier than necessary.

Light designs are important in aerospace technology. Lighter designs might be possible if the designs could be based on the statistical matching of the available strength to the imposed stress. The NASA-Lewis Research Center in conjunction with The University of Arizona has undertaken the task of developing a mechanical design methodology of this type. The design methodology being developed under this research is trying to reduce the number of these modifying factors by improved laboratory testing methods. It is dealing with important design variables as statistical distributions rather than single valued estimates. It also makes provisions for designing an a priori reliability into a mechanical component.

In order to work out this methodology the following information is required:

(1) Statistical strength distributions for a selected material.

(2) Statistical stress variations in a loading configuration.

(3) Statistical methods for relating stress to strength.

This paper discusses the analytical, experimental and application status accomplished during the first phase of this research.

A typical rotating part design problem is given in this paper to illustrate the conventional design technique. The same problem is reworked using the reliability design methodology developed during Phase I of this research to see if weight savings can be accomplished.

ANALYSIS

DETERMINE THE FAILURE GOVERNING STRENGTH - In the engineering application of the design-by-reliability methodology, one of the most difficult tasks is to determine the actual strength and stress distributions involved for a specific material in a specific application. Two methods for determining distribution parameters for material strengths have been developed thus far: (1) Random variable functions; and (2) Direct experimentation. Much work remains to be done in this area.

Random Variable Functions - The method of random variable functions as applied to material strengths is concerned with finding a statistical function which accurately represents the material's strength. Considerable research effort has been expended by Kececioglu (1)* and others to determine which distribution defines experimental fatigue data best. Distributions which have been studied are the normal, lognormal and Weibull.

A procedure to convert readily available tensile strength data, such as that given in Table A-3 of Ref. (14, p. 600), to statistical functions which accurately represent the material's endurance strength would be very useful. This table gives yield strength, S_y, and ultimate tensile strength, S_{ut}. In many cases where endurance strength distribution data for AISI steels are not available, use can be made of the common practice, based on the work of Lipson and Juvinall (2, p. 162), to obtain the mean estimate for endurance limit for steels as:

$$\mu_{S_e'} = 0.50\ S_{ut};\ \text{when}\ \ S_{ut} \leqq 200\ \text{kpsi}$$

$$\mu_{S_e'} = 100\ \text{kpsi};\ \text{when}\ \ S_{ut} \geqq 200\ \text{kpsi}$$

The standard deviation for the normal distribution can be obtained by making use of a conclusion based on Kececioglu's and others' work (3, 4). These investigators have found that a standard deviation of 7 percent of the unmodified endurance limit can be used if little or no test data is

*Numbers in parentheses designate References at end of paper.

available. The final expression for S_e becomes $(\mu_{S_e'}, 0.07\ S_e')$.

Marin's work (5, p. 127) can be extended to obtain a machined part's distributional endurance limit S_e by considering each parameter in Eq. (1) below as a random variable.

$$S_e = k_a k_b k_c k_d k_e k_f S_e' \tag{1}$$

Products of random variables can be calculated by various methods (6,7). A later section gives an example calculation of this type for the case of k_a, k_b and k_e unit normal modifiers, k_c is included by a different method and S_e' is based on experimental data rather than obtained from tables as explained previously.

A second procedure for determining the distribution of the fatigue strength of materials has been worked out using Monte Carlo Techniques. This procedure also requires knowledge of the constants k_a through k_f with an estimate of S_e' from existing data. Digital computing techniques have been developed to calculate the first four moments of a statistic formed by the product of 1000 draws from the normal distribution of the known variables (8, 9, 10 and 11). Results obtained from the Monte Carlo computer calculations compare within 5 percent of those obtained by the Algebra of Normal Functions Method when the coefficients of variation are small. Therefore, this latter method provides a simple acceptable method for estimating strength distributions under these conditions.

Direct Experimentation - In some cases it may not be possible to estimate the distribution parameters for the fatigue strength of a material. This almost always is due to a lack of statistically meaningful test data and/or engineering experience with a particular material. A laboratory test procedure by which fatigue strength for different materials could be obtained is desired.

The first requirement of the test procedure would be to design suitable test specimens. Each test specimen must be carefully prepared and inspected to assure that it reproduces as many of the essential features of the part application as possible. Many times it is necessary to scale various test specimen parameters making an effort to retain the same stresses, stress concentration factors, stress ratios and material conditions. A statistically meaningful number of these test specimens is also required.

A machine to test these specimens under conditions similar to the part application is also needed. The fatigue testing machines used in this research effort are explained later. Groups of specimens are tested to failure at fixed alternating stress levels and stress ratios. Fig. 1 shows the distributional S-N diagrams obtained from this testing. Five alternating stress levels and an endurance strength stress level are determined. The specimens are subjected to fixed magnitudes of reversed bending and steady torque to keep the stress ratio (alternating/mean), r_s, at fixed values. Fig. 1(a) is for $\bar{r}_s = \infty$ and Fig. 1(b) is for $\bar{r}_s = 0.825$ for the cycles-to-failure and for $\bar{r}_s = 1.03$ for endurance.

The cycles-to-failure distributions for a particular stress ratio can be converted to the material's strength distribution at any desired cycles of life. The cumulative histogram is formed to show the percentage of specimens failing in each stress cell for a fixed cycle-life value. From this cumulative histogram, the material strength distribution can be found by using statistical methods. The resulting statistical distribution is the material's strength for a fixed number of cycles of life and stress ratio. A number of strength distributions may then be translated onto the Goodman diagram, such as shown in Fig. 2, and the experimental failure governing strength distribution surface may thus be obtained. Fig. 2 shows a theoretical Goodman complex-fatigue strength surface. It seems that to establish an acceptable Goodman fatigue strength surface, at least four distributions ($\bar{r}_s = \infty$, 3.0, 0.70 and 0.30, at a specified life cycle, $K_L = 10^n$ cycles) would be required. This coupled with the static ultimate tensile strength distribution, or at $\bar{r}_s = 0$, gives five sections on the contour to define the strength surface. The contours shown in Figs. 1 and 2 are for the two parameter normal distribution. A detailed task is being considered to determine if other distribustions would fit this data better.

DETERMINE THE FAILURE GOVERNING STRESS - The problem of determining the failure governing stress is concerned with determining which failure governing theory applies best for the material, loads, dimensions, stress concentrations and the like for the part in service. Rotating shafts can be analyzed using the von Mises Hencky failure criterion (8, 12). This criterion is applicable to ductile steels (8, pp. 152-154), and is theoretically valid for the elastic region only. It is recognized that the steel used in this research fits the ductile requirement in the unnotched form. The elastic region constrain is not met either, as the specimens are tested to fracture. The justifications for its use are that (1) correlation does exist between the results predicted by it and the experimental results (13, pp. 38-41) and; (2) it is the only theoretically formulated criterion which explicitly incorporates all stress components involved.

Fig. 3 shows the rotating shaft surface stress element for the two-dimensional case. The equations for components of bending, radial and shear stresses are also given in this figure. Using the stress components as defined in the stress diagram, and according to the von Mises-Hencky criterion (14, p. 154), the failure governing alternating stress, s_a, is given by

$$s_a = \left(\bar{\sigma}_{xa}^2 - \bar{\sigma}_{xa}\bar{\sigma}_{ya} + \bar{\sigma}_{ya}^2 + 3\bar{\tau}_{xya}^2\right)^{1/2}$$

and the failure governing stress $\bar{s}_m$ by

$$s_m = \left(\bar{\sigma}_{xm}^2 - \bar{\sigma}_{xm}\bar{\sigma}_{ym} + \bar{\sigma}_{ym}^2 + 3\bar{\tau}_{xym}^2\right)^{1/2}$$

The test specimens studied in this research are explained later. For this specimen configuration, the following conditions apply:

$$s_a = \sigma_{xa} = \frac{M}{\frac{I}{C}} = \frac{(\mu_M, \sigma_M)}{(0.098\ \bar{d}^3,\ 0.000244\ \bar{d}^3)} \qquad (2)$$

$$s_m = (3)^{1/2}\tau_{xym} = \frac{(3)^{1/2}T}{\frac{J}{C}}$$

$$= \frac{1.73(\mu_T, \sigma_T)}{(0.196\ \bar{d}^3,\ 0.000488\ \bar{d}^3)} \qquad (3)$$

A detailed example of the use of similar equations for a specific case is given later.

The magnitudes of the corresponding failure governing strength (stress-to-failure) components can be obtained from the three-dimensional Goodman diagram by using graphical construction techniques, shown in Fig. 10. Fig. 10 shows a probabilistic, three-dimensional Goodman diagram. The top portion of the Goodman strength surface is obtained by drawing three straight lines to represent a distributional (normal) surface. The mean estimate loci are drawn as solid lines and the plus or minus three times the standard deviation loci are drawn as dashed lines. These lines extend through experimental data points or from an ordinate of $(\mu_{S_e}, \sigma_{S_e})$ to the junction of $(\mu_{S_y}, \sigma_{S_y})$ and $(\mu_{S_{ut}}, \sigma_{S_{ut}})$. A second stress plane is needed to define S_a and S_m. This surface is defined by the probability density axis and the line passing through the origin subtending an angle of θ_2 with the abcissa. The angle θ_2 is given by the application stress ratio:

$$r_s = \frac{s_a}{s_m} = \frac{\mu_{s_a}}{\mu_{s_m}}$$

$$\theta_2 = \tan^{-1}\frac{\mu_{s_a}}{\mu_{s_m}}$$

The intersection of this line with the mean Goodman line defines S_a and S_m.

BRIDGE THE GAP BY RELIABILITY THEORY - The reliability of a component can be determined from the basic concept that a no-failure probability exists when a given strength, S, value is not exceeded by stress, s. The probability that a stress of value s exists in the interval $s_1 - ds/2$ to $s_1 + ds/2$ is equal to the area of the element ds, or to A_1 in Fig. 4, or

$$P\left(s_1 - \frac{ds}{2} \leq s \leq s_1 + \frac{ds}{2}\right) = f(s_1)ds = A_1$$

The probability of strength exceeding s_1 is equal to the shaded area A_2, or

$$P(S > s_1) = \int_{s_1}^{\infty} f(S)dS = A_2$$

The probability of no failure, that is, the reliability, at s_1 is the product of these two probabilities, or

$$dR = f(s_1)ds \times \int_{s_1}^{\infty} f(S)dS$$

The component reliability would then be all probabilities of strength being greater than all possible values of stress (13,15) or

$$R = \int dR = \int_{-\infty}^{\infty} f(s)\left[\int_{s}^{\infty} f(S)dS\right]ds \quad (4)$$

The reliability can also be written as

$$R = \int_{-\infty}^{\infty} f(S)\left[\int_{-\infty}^{S} f(s)ds\right]dS \quad (5)$$

Eqs. (4) and (5) can now be used to calculate the reliability of any component whose f(s) and f(S) are known. These equations carry limits of integration applicable to distributions defined over the interval from $-\infty$ to $+\infty$. For functions defined in different intervals these limits should be replaced by the lowest and highest values that can be used. Convolution integral techniques, transform methods, or a computer program utilizing Simpson's Rule (13, pp. 177-197) may be employed to evaluate these equations and thereby determine reliability.

If the density functions f(s) and f(S), representing the stress and strength distributions, respectively, are Gaussian or normal, as shown in Fig. 5, then they may be expressed as

$$f(s) = \frac{1}{\sigma_s\sqrt{2\pi}} e^{-1/2[(s-\bar{s})/\sigma_s]^2}$$

$$f(S) = \frac{1}{\sigma_S\sqrt{2\pi}} e^{-1/2[(S-\bar{S})/\sigma_S]^2}$$

Reliability is given by all probabilities that strength is in excess of stress or that $S - s > 0$. Using the designation $\xi = S - s$, reliability is given by all of the probabilities that $\xi > 0$. Let $h(\xi)$ be defined as the difference distribution of f(S) and f(s), and as f(S) and f(s) are normally distributed, then $h(\xi)$ is normally distributed also (16, pp. 215-216) and is expressed by

$$h(\xi) = \frac{1}{\sigma_\xi\sqrt{2\pi}} e^{-1/2[(\xi-\bar{\xi})/\sigma_\xi]^2}$$

where $\bar{\xi} = \bar{S} - \bar{s}$

and $\sigma_\xi = (\sigma_S^2 + \sigma_s^2)^{1/2}$

Reliability would then be given by all probabilities of ξ being a positive value, hence

$$R = \frac{1}{\sigma_\xi\sqrt{2\pi}} \int_0^{+\infty} e^{-1/2[(\xi-\bar{\xi})/\sigma_\xi]^2} d\xi$$

The relationship between $h(\xi)$ and the standardized normal distribution can be utilized to evaluate the above integral. The transformation relating ξ and the standardized variable z may be used which is

$$z = \frac{\xi - \bar{\xi}}{\sigma_\xi}$$

The new limits of the integrand are

for $\zeta = 0, \; z = \frac{0 - \bar{\zeta}}{\sigma_\zeta} = - \frac{\bar{\zeta}}{\sigma_\zeta}$

and for $\zeta = +\infty, \; z = \frac{+\infty - \bar{\zeta}}{\sigma_\zeta} = +\infty$

also $d\zeta = \sigma_\zeta \, dz$

If these conditions are substituted into the reliability equation, the following result is obtained:

$$R = \int_{-\bar{\zeta}/\sigma_\zeta}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-1/2 z^2} dz \qquad (6)$$

Consequently, the reliability of a component is given by the area under the standardized normal density function from the value of

$$z = - \frac{\bar{\zeta}}{\sigma_\zeta} \text{ to } z = +\infty$$

The value of this area may be obtained from the tables of areas under the standardized normal density function, available in many references.

APPARATUS AND PROCEDURE

The combined bending-torsion fatigue-reliability research machines are designed to simulate a shaft in service. The objective of the immediate research program is to examine the fatigue life of specimens made of SAE 4340 steel under combined loadings. The machine is designed to subject the specimens to a predetermined range of combined reverse bending and steady torque. A number of specimens are tested under this range of conditions to obtain the statistical distribution of the material's strength.

Each fatigue machine consists of a two-section, rotating shaft with a test specimen locked in the center, as shown in Fig. 6. The horizontal shaft is coupled at each end to allow for relatively free deflection when the specimen is loaded. A $7\frac{1}{2}$ horsepower, 1800 rpm motor powers the shaft. The bending load is applied to the specimen by means of two yokes, one each on two bearings located symmetrically about the specimen on two commercial tool holders. Below the shaft, the yokes are connected by a horizontal link, which concentrates the load at a single vertical link in the center. The vertical link is then connected to either a long or a short loading lever arm. These loading arms make possible the application of a great range of bending stresses in the specimen groove, by means of pan weights applied at the end of the loading arm. One lb of pan weight is equal to approximately 2000 psi bending stress in the groove. The torque is applied by means of a commercial Infinit-Indexer which is located on the back shaft of the machine. Table 1 summarizes the operational specifications for the complex fatigue research machines. The machine is capable of producing, holding and transmitting to the rotating specimen steady torques of up to 5400 lb-in. and reversed bending moments of up to 3450 in.-lb. Specimens of diameters up to 1 in. can be tested in this machine by changing the collets in the specimen holder.

The SNAP-8 turbine shaft design specifications were used as a guide to make the research data directly applicable to Aerospace problems. The American Society of Testing Materials Standards were used as a guide for testing (17). Fig. 7 shows the notched test specimen details. Bar stock pieces 6 in. long where carefully machined to the dimensions and finish specified in Fig. 7(a). Each specimen has a ground-in circumferential groove and keyway. The reverse bending moment M, torque T, bending stress σ_{xa}, and shear stress τ_{xym}, are also defined in Fig. 7(a).

Each specimen is installed in the test machine and the instrumentation is checked for zero adjustment and calibration. Appropriate bending moment is applied to the specimen by putting weights on the load pan. Torque is applied to the specimen through the Infinit-Indexer with a suitable wrench. The timing clock is set to zero and the machine is started. When the specimen fractures a microswitch stops the clock and the machine; See Fig. 7(b) for the actual specimen before and after fracture.

EXPERIMENTAL RESULTS

A total of 396 specimens have been tested. Only 238 of these specimens yielded useful results: 150 specimens yielded cycles-to-failure data, 68 specimens yielded endurance strength data, and 20 specimens yielded static strength data. The remaining 158 specimens were not included in the reported test data because the machine test conditions varied over too wide a range during the test, or the specimen broke at the keyway rather than in the groove.

Tables 1 to 5 present the data obtained from the laboratory testing machines. Table 2 shows the cycles-to-failure data for stress ratios of ∞ and 0.825. Tables 3 and 5 show the notched and unnotched tensile strength data for a stress ratio of zero. Table 5 shows the endurance strength data for stress ratios of ∞ and 1.03. Table 6 shows a summary of the best fit equations for the cycles-to-failure data as given in Tables 2 and 4.

DESIGN EXAMPLES

A typical rotating part design problem is explained below. The inner shaft for an alternator rotor (see Fig. 8) is designed using modifying factors to reduce the material's endurance limit. The shaft diameter following this design procedure came out to be 5/8 in.

The inner shaft for an alternator rotor is redesigned using the probabilistic design methodology. The shaft diameter following the distributional design procedure came out to be 1/2 in. This decrease in shaft diameter over the original requirement results in about a 38 percent decrease in weight of material required to fabricate this part. None of the original design requirements were reduced. These requirements were met by matching material to an application through the use of statistical methods.

CONVENTIONAL SHAFT DESIGN - The inner shaft shown in Fig. 8 is being designed to support an alternator rotor. The shaft is made of AISI 4340 steel, heat treated to 340/380 Brinell hardness with a machined finish. The rotor subjects the shaft to a radial bending load of 136.8 lb and a torque of 1000 lb-in. when delivering full load electrical power. The spline drive can exert a maximum misalignment radial force of 3 lb. The hot gas turbine drive will raise the shaft temperature to 500° F. The shaft rotates at 12,000 rpm and is to have a life of greater than 10^8 cycles corresponding to a 0.999 component reliability. What should the shaft diameter be?

There are various methods that can be used for machine design problems in which the shaft diameter is unknown. For aerospace applications life and reliability are very important considerations. Shigley (14, p. 482) has shown that under these conditions it is wise to use the von Mises-Hencky-Goodman method. This method requires knowledge of material strengths and application stresses.

The example design problem specifies that this shaft is to be made of AISI 4340 steel, heat treated to 340/380 Brinell hardness. Table A-3 (14, p. 600) gives the tensile properties for 1 in. round, drawn 1000° F condition, 377 Brinell hardness material as $S_y = 160$ kpsi and $S_{ut} = 187$ kpsi.

Following Joseph Marin's suggestion (5, p. 127) the endurance limit will be reduced by modifying factors. These modifying factors attempt to account for the differences between the machined part's application environment and the endurance limit defined by laboratory testing. A machined part's endurance limit can be calculated by using Eq. (7) below:

$$S_e = k_a k_b k_c k_d k_e k_f S_e' \qquad (7)$$

The modifying factors will be explained later. In many cases, when endurance strength data for AISI steels are not readily available, use can be made of the common practice, based on the work of Lipson and Juvinall (2), to estimate endurance limits for steels as:

$$S_e' = 0.50\ S_{ut};\ \text{when}\ \ S_{ut} \leq 200\ \text{kpsi}$$

For AISI 4340 steel

$$S_e' = 0.50 \times 1.87 \times 10^5\ \text{psi} = 93.5\ \text{kpsi}$$

The fatigue strength modifying factors for this problem can be obtained from existing literature. The surface finish factor, k_a, (14, Fig. 5-26, p. 167) is 0.65 for $S_{ut}=187$ kpsi. The size factor, k_b, (18, p. 119) is 0.85 for specimens up to 2 in. The reliability factor, k_c, (14, Eq. 5-14, p. 169) is

$$k_c = 1 - 0.08\,D = 1 - 0.248 = 0.752$$

The standard deviation multiplication factor D is 3.1 for a survival rate of 99.9 percent. This factor begins to recognize that many design parameters are not fixed, discrete integers but are more correctly represented by probability density functions of various kinds. The probabilistic design methodology explained later attempts to develop this concept still further. The temperature factor k_d is somewhat harder to estimate as only meager data exists. Curves of mean stress versus alternating stress for A 286 alloy at 70° F and 1000° F are available (19, Fig. 2-36, p. 106). Each curve represents failure by rupture in 500 hr at the temperature indicated. Interpolating from the data given in this figure $k_d = 0.867$.

The inner shaft, as shown in Fig. 8(a), would fail at the shoulder fillet just to the left of the keyway should the rotor bind for some reason. The shaft is driven by the hot gas turbine through a spline coupling. At full electrical load the shaft carries 1000 lb-in. of torque. The direction of torque is given by the arrow in the load diagram. The bending moment diagram is given in Fig. 8(b). The shoulder just to the left of the keyway is the most dangerous section because of stress concentration. The clockwise bending moment here calculates out to be 126 in.-lb.

The minimum theoretical stress concentration factors at this shoulder are (14, Fig. A-12-8, 9, pp. 616-617)

$$K_{s_{max}} = 1.11 \quad \text{and} \quad K_{b_{max}} = 1.37$$

for a shaft range of 1/2 to 5/8 in. diameter and the 1/8 in. shoulder radius. What to do when stress concentration is caused by three effects has not been investigated much, but it seems reasonable that the combined effect should be no greater than the product of each factor. Therefore,

$$K_{t_{comb\ max}} = 1.11 \times 1.37 = \underline{1.52}$$

Notch sensitivity, q, (14. Fig. 5-27, p. 171) based on data (20, pp. 296, 298) for a notch radius of 0.125 in. and $S_{ut} = 187$ kpsi is 0.90. The maximum fatigue stress concentration factor $K_{f_{max}}$ is calculated by

$$K_{f_{max}} = 1 + q(K_{t_{max}} - 1) = 1 + 0.90(1.52-1) = 1.47$$

Finally, the minimum stress concentration modifying factor $k_{e_{min}}$ is given by

$$k_{e_{min}} = \frac{1}{K_{f_{max}}} = 0.680$$

The miscellaneous modifying factor, such as residual stress, directional characteristics, internal defects, corrosion, plating are all taken to be unity for this example case.*

Using Eq. (7), the modified endurance limit for this case would be

$$S_e = 0.65 \times 0.85 \times 0.752 \times 0.867 \times 0.680 \times 1.0 \times 9.35 \times 10^4$$

$$= 2.28 \times 10^4 \text{ psi}$$

The bending moment M at the shoulder is 126 in.-lb. The bending stress, σ_{xa}, at the shoulder is calculated as follows:

$$\sigma_{xa} = \frac{M}{\frac{I}{C}} = \frac{126 \times 32}{\pi d^3} = \frac{1.28 \times 10^3}{d^3}$$

The torque T is constant over the shaft length at 1000 lb-in. The torsional stress

*Some of the references cited in this report discuss methods for handling miscellaneous modifying factors - Ref. (14) being especially useful.

τ_{xym} in the unknown diameter section of the shaft is:

$$\tau_{xym} = \frac{T}{\frac{J}{C}} = \frac{1000\times 16}{\pi d^3} = \frac{5.1\times 10^3}{d^3},$$

neglecting the contribution of the alternating shear stress due to the vertical shear at the critical section.

Analysis has been made of the various failure theories suitable for defining stress components under these conditions (8, p. 37, 12, p. 479). The von Mises-Hencky failure theory has gained recognition as it seems to explain laboratory observations well. The von Mises-Hencky stress components s_a and s_m are given by

$$s_m = \left(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2 + 3\tau_{xym}^2\right)^{1/2}$$

$$s_a = \left(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2 + 3\tau_{xya}^2\right)^{1/2}$$

For this problem, a reasonable model sets σ_{xm}, σ_{ym}, σ_{ya} and τ_{xya} all equal to zero. Therefore,

$$s_m = \sqrt{3}\,\tau_{xym} = \frac{8.83\times 10^3}{d^3}$$

$$s_a = \sigma_{xa} = \frac{1.28\times 10^3}{d^3}$$

$$r_s = \frac{s_a}{s_m} = 0.144$$

A conventional Goodman diagram can be prepared from this data. Alternating and mean stress are plotted as ordinate and abscissa, respectively, in Fig. 9. Defining stress-to-failure as material strength makes it possible to use the Goodman boundary to represent the strength of this material as modified by environmental factors. The top portion of the Goodman boundary is obtained by drawing a straight line between two points - ordinate ($S_e = 0,\ 2.28\times 10^4$) and abscissa ($S_{ut} = 1.87\times 10^5,\ 0$). The right boundary is defined by a point ($S_y = 1.60\times 10^5,\ 0$) and an angle ($\tan^{-1} 6 = 80.5^o$) as the abscissa scale was choosen to be six times the ordinate scale to improve accuracy. A third straight line is constructed to define S_a and S_m for this case. This line is defined by a point and an angle - (0,0) and ($\tan^{-1} 0.865 = 40.8^o$). The intersection of Line 1 and Line 3 define the point $S_a = 12.3\times 10^3$ psi and $S_m = 85.0\times 10^3$ psi.

Conventional design practice presently is using a margin of safety of at least 100 percent to insure a safe design. This requires that σ_{xa} meet the following constraint:

$$\sigma_{xa} \leq \frac{S_a}{2.0} = \frac{12.3\times 10^3}{2.0}$$

Substituting values gives

$$\frac{1.28\times 10^3}{d^3} \leq 6.15\times 10^3$$

$$d^3 \geq \frac{1.28\times 10^3}{6.15\times 10^3} = 0.209$$

or

$$d \geq 0.590 \text{ in.}$$

Based on this results use a shaft diameter of 19/32 in. The results for the conventional design method are tabulated in Table 7 for purposes of comparison with the probabilistic design method.

PROBABILISTIC SHAFT DESIGN - The alternator shaft designed in the previous section shall be redesigned using the probabilistic methodology. The probabilistic design methodology being developed under Grant NGR 03-002-044 is based on Marin's technique using a modified form of the von Mises-Hencky-Goodman diagram. The primary change is to consider the necessary design parameters as probability density functions and the Goodman diagram as surfaces. Considerable laboratory testing is also being done to define material strength surfaces in this form.

The example design problem specified that the alternator shaft is to be made of AISI 4340 steel, heat-treated to 340/380 Brinell hardness. The laboratory test program explained in the main text of this report gives test data for SAE 4340 steel which is similar to AISI 4340, condition C4, per MIL-S-5000B, heat treated to Rockwell C 35/40 per MIL-H-6875, with minimum tempering temperature of 1000° F, inspected according to MIL-I-6868. These tests showed that $S^{*}_{ut} = (2.56\times10^5;\ 2.5\times10^3)$ for notched specimens and $S_{ut} = (1.78\times10^5;\ 2.5\times10^3)$ for unnotched specimens (see Tables 3 and 4). This mean estimate is 0.96 times that given in Table A-3 of (14, p. 600). The unnotched yield strength given in Table 5 is $S_y = (1.71\times10^5;\ 3\times10^3)$ which agrees with the value given in Table A-3 of (14) fairly well (10.7 percent) and serves as a check on the test program validity.

Since the laboratory test program is generating data for endurance limits under combined conditions of reverse bending and steady torque for various stress ratios, not as many modifying factors are required to determine the parts application endurance limit. The test specimens were carefully machined to control surface finish so k_a has been included in the test program. These tests were run on round specimens with D = 0.735 in., d = 0.4975 in. and r = 0.150 in.; therefore, k_b is already included. The reliability factor k_c is introduced differently based on the overlap range of stress into strength as explained in the body of this report. All of the test specimens run on this project so far have been in a laboratory ambient temperature of 75° $\pm 5^{\circ}$ F; therefore, an estimate of the modifying factor for temperature k_d is required. One value for $k_d = 0.867$ can be obtained from (19), Fig. 2-36, p. 106 for unnotched A 286 alloy steel. A second value for $k_d = 0.921$ can be obtained from (8), Fig. 1.20, p. 46 based on (20) for unnotched, 1.8 percent nickel, 0.8 percent chromium alloy steel. To obtain a normal probability function, define the range of k_d as 0.867 to 0.921. The mean estimate of 0.894 with a standard deviation of 0.009 for this range. The difference in k_e for this shaft and the test specimens is negligible. All other modifying factors are assumed to be unity.

*Normal distribution notation (mean, standard deviation).

Table 5 shows that $S'_e = (5.65\times10^4;\ 3.2\times10^3$ psi) for this material for $r_s = 1.03$ and that there is little change for $r_s = \infty$.

Using the algebra of normal functions and Eq. (7), the modified endurance limit for this case can be calculated as follows:

$$S_e = (0.894;\ 0.009)(5.65\times10^4;\ 3.2\times10^3)$$

$$S_e = (5.08\times10^4;\ 2.75\times10^3)\ \text{psi}$$

The bending moment, M, at the shoulder was estimated to be (126; 11.4) in.-lb. The bending stress, σ_{xa}, at the shoulder is calculated as follows

$$\sigma_{xa} = \frac{M}{\frac{I}{C}} = \frac{(126;\ 11.4)}{(0.098\ \bar{d}^3;\ 0.000735\ \bar{d}^3)} =$$

$$= \left(\frac{1.28\times10^3}{\bar{d}^3};\ \frac{116.4}{\bar{d}^3}\right) \qquad (8)$$

The distance to the outer fibers C is given by $(0.5\ \bar{d};\ 0.0025\ \bar{d})$. Standard deviation for this estimate is based on a manufacturer's guaranteed tolerance, Δd, of $0.015\ \bar{d}$. The moment of inertia, I, is given by $(0.049\ \bar{d}^4;\ 0.00295\ \bar{d}^4)$ for this configuration. The ratio I/C is given by $(0.098\ \bar{d}^3;\ 0.000735\ \bar{d}^3)$. Substituting these values yields the solution for bending stress given in Eq. (8).

The shearing moment, T, transmitted by this shaft is given to be (1000; 80) lb-in. The torsional stress τ_{xym} at the shoulder is calculated as follows:

$$\tau_{xym} = \frac{T}{\frac{J}{C}} = \frac{(1000;\ 80)}{(0.196\ \bar{d}^3;\ 0.00147\ \bar{d}^3)}$$

$$= \left(\frac{5090}{\bar{d}^3};\ \frac{409}{\bar{d}^3}\right) \qquad (9)$$

The von Mises-Hencky stress components are given as

$$s_a = \left(\frac{1.28\times10^3}{\bar{d}^3} ; \frac{116.4}{\bar{d}^3} \right)$$

using Eqs. (2) and (8), and

$$s_m = \left(\frac{8.83\times10^3}{\bar{d}^3} ; \frac{709}{\bar{d}^3} \right)$$

using Eqs. (3) and (9).

$$\bar{r}_s = \frac{s_a}{s_m} = 0.144$$

The above two stress distributions should be synthesized into the failure governing stress distribution $f(s_f)$, along $\bar{r}_s$ in Fig. 10. Along $\bar{r}_s$

$$s_f = \left(s_a^2 + s_m^2 \right)^{1/2} ,$$

hence using the partial derivatives method (22, pp. 228-235), and the parameters for $f(s_a)$ and $f(s_m)$ given above, we get

$$\bar{s}_f = \left[\left(\frac{1.28\times10^3}{\bar{d}^3} \right)^2 + \left(\frac{8.83\times10^3}{\bar{d}^3} \right)^2 \right]^{1/2}$$

and

$$\sigma_{s_f} \cong \left[\left(\frac{\partial s_f}{\partial s_a} \right)^2 \sigma_{s_a}^2 + \left(\frac{\partial s_f}{\partial s_m} \right)^2 \sigma_{s_m}^2 \right]^{1/2}$$

where

$$\frac{\partial s_f}{\partial s_a} = \frac{\bar{s}_a}{\bar{s}_f} \text{ and } \frac{\partial s_f}{\partial s_m} = \frac{\bar{s}_m}{\bar{s}_f} .$$

Consequently,

$$\sigma_{s_f} \cong \left(\left\{ \frac{\frac{1.28\times10^3}{\bar{d}^3}}{\left[\left(\frac{1.28\times10^3}{\bar{d}^3} \right)^2 + \left(\frac{8.83\times10^3}{\bar{d}^3} \right)^2 \right]^{1/2}} \right\}^2 \left(\frac{116.4}{\bar{d}^3} \right)^2 + \left\{ \frac{\frac{8.83\times10^3}{\bar{d}^3}}{\left[\left(\frac{1.28\times10^3}{\bar{d}^3} \right)^2 + \left(\frac{8.83\times10^3}{\bar{d}^3} \right)^2 \right]^{1/2}} \right\}^2 \left(\frac{709}{\bar{d}^3} \right)^2 \right)^{1/2}$$

Thus $f(s_f)$ should be coupled with $f(s_f)$ to determine the shaft diameter with the specified reliability. Consequently $f(s_f)$ should be determined next. A probabilistic Goodman diagram can be prepared from the research data. Fig. 10 is the resultant diagram for this case. The top portion of the Goodman surface is obtained by drawing three straight lines to represent a normal surface. The mean estimate loci is drawn as a solid line - ordinate $(5.08\times10^4;\ 0)$ and $\bar{r}_s = 1.03$ at $(5.08\times10^4;\ 5.4\times10^4)$.* From here on data from the combined stress fatigue reliability research machines has not been reduced as yet. The middle portion of the Goodman surface connects the unnotched** ultimate strength $(1.78\times10^5;\ 2.5\times10^3)$ to the $\bar{r}_s = 1.03$ points. The right hand boundary is defined by the surface boundary $(1.71\times10^5;\ 3\times10^3)$ and the angle $(\tan^{-1} 2 = 63.5^\circ)$ as the abscissa scale was chosen to be two times the ordinate scale to improve accuracy. A second surface is constructed to define S_a and S_m for this case. This surface is defined by a point $(0;\ 0)$ and three lines passing through the origin subtending an angle of $(16.1\pm1^\circ)$ with the abscissa.

*This point is based on the experimental data given in Table 5.

**This is the currently accepted practice. It's validity shall be established at the conclusion of the present phase of this research.

Neglecting the effect of the very small variability in $\overline{r}_s$, the intersection of the strength surface with the $\overline{r}_s$ line defines S_a to be approximately (1.91×10^4; 1.10×10^3) and S_m to be approximately (1.31×10^5; 1.21×10^3). Combining these two distributions gives a failure governing strength distribution along $\overline{r}_s$ defined as (1.33×10^5; 1.23×10^3).

The mean and standard deviation of $f(s_f)$ and $f(S_f)$ have thus been determined. If it is assumed that these distributions are normal, then the parameters of these distributions are known and are those determined previously. The amount by which these two normal distributions overlap has been specified by a part reliability requirement of 0.999. Eq. (6) may be used to approximate the reliability of this complex case. Then for $R = 0.999$, $-\xi/\sigma_\xi$ should be 3.09 (21, Table 3, p. 351) or since

$$-\frac{\overline{\xi}}{\sigma_\xi} = -\frac{\mu_{S_f} - \mu_{s_f}}{\left(\sigma_{S_f}^2 + \sigma_{s_f}^2\right)^{1/2}}, \tag{10}$$

substituting previously obtained values into Eq. (10) and solving for d yields

$$\overline{d}^6 - 0.134284\,\overline{d}^3 + 0.00412 = 0$$

Solving for $\overline{d}$ gives a $\overline{d}$ of 0.438 in. Use a shaft diameter of 15/32 in. It must be pointed out that the portion of the Goodman diagram in Fig. 10 used to determine $f(s_f)$ is conservative hence this shaft's weight may be reduced further.

The results for the probabilistic design method are tabulated in Table 7. Three modifying factors are not required as the experimental data includes these factors. An a priori reliabililty is designed into this part by making use of difference statistics. The probabilistic design methodology results in better than a 38 percent savings in materials for this case. Furthermore the design integrity is known to have a reliability of 0.999, such that on the average no more than one in a 1000 such shafts will fail while functioning during their designed for mission.

CONCLUDING REMARKS

The Lewis Research Center in conjunction with The University of Arizona has undertaken the task of developing a mechanical design methodology to statistically match the available strength to the imposed stress. Analytical and experimental methods have been developed to reduce the number of modifying factors necessary to determine a material's endurance limit. Important design variables are handled as statistical distributions. Provisions for designing an a priori reliability into a mechanical component are explained. In order to work this methodology out completely for one case, methods for determining (1) statistical strength distributions for SAE 4340 steel; (2) statistical stress variations for a rotating shaft subject to reverse bending and torque; and (3) statistical methods for relating strength to stress are given in this paper.

The sample mechanical design problems given in this paper show that weight savings are possible when the new design methodology is compared with the existing von Mises-Hencky-Goodman design procedure.

There are some serious problems in trying to put the probabilistic design methodology into broad usage. The most serious of these problems is the gross lack of distributional data for material strengths. A second problem is the lack of loading distributions and combinations of stress distributions. Experimental test programs are currently under way to obtain more data. These efforts are directed toward alloy and carbon steels suitable for use in the design of machined parts subjected to fatigue.

A third problem is working with the tails of statistical functions. For valid work to be accomplished in this region, the statistical functions must be based on an unbiased sample of the population. One of the important tasks defined in Phase II of this research is to determine the effect of sample size on the discrimination ability of selecting the best distribution from among various distributions that fit the experimental data.

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Table 1 - Operational Specifications for the Strength Distribution Research Machines

Test Machine[a]	Accomodate a specimen rotating at 1800 rpm; produce and hold a steady torque and a reversed bending moment; holding chuck 1.0 in. diameter maximum; simple design employing "off-the-shelf" components.
Loading Mechanism for Steady Torque:	Simple device to produce, hold, and transmit desired steady torque of 5400 lb-in. to test specimen.
Loading Mechanism for Reversed Bending:	Simple device to produce a reversed bending moment of 3450 in.-lb while specimen is rotating.
Test Specimen:	SAE 4340 steel, Condition C-4; MIL-S-5000B, certification of chemical and physical properties; uniform quality, same heat and processing, heat treat to Rockwell "C" 35/40 as per MIL-H-6875 with minimum tempering temperature of 1000° F; inspection as per MIL-I-6868; D = 0.735 in., d = 0.500 in., r = 0.150 in.; $K_b = 1.45$; $K_s = 1.22$, with 1 in. × 1/8 in. × 1/8 in. keyways.
Instrumentation:	
Strain gages	To obtain dynamic and static strain measurements in bending and torsion.
Channels	To handle at least 6 sets of strain gage outputs simultaneously.
Slip rings	To transfer strain gage data to amplifier while specimen is rotating.
Amplifier	To amplify static and dynamic output from strain gages.
Recorder	To produce a permanent record of amplified strain gage outputs.

[a]Three such machines have been built to date and are being used to obtain distributional complex fatigue data for the steel specimens specified above.

Table 2 - Cycles-To-Failure Fatigue Data Material: SAE 4340 Steel, Rockwell 35/40 MIL-S-5000B, Condition C4

r_s(a)	∞					0.825			
μr_s	$s_m = 0$					0.876	0.800	0.818	0.805
σr_s						0.226	0.082	0.070	0.056
K_V						3.1	8.0	6.2	4.3
s_a	150,000	120,000	100,000	80,000	60,000	120,000	100,000	80,000	75,000
μs_a(b)	154.0	114.0	98.0	81.5	73.0	111.0	91.5	75.5	65.0
σs_a(c)	1.5	0.0	2.7	0.9	1.8	1.3	6.7	3.1	3.9
K_V(d)	1.0	0.6	2.8	1.1	2.5	1.2	7.3	4.2	6.0
n									
1	1,550	7,000	16,250	60,000	93,300	5,400	12,950	39,550	99,400
2	1,950	7,600	16,500	64,850	103,400	5,650	13,050	43,950	100,900
3	2,450	7,700	16,900	65,000	124,150	5,900	13,350	46,450	101,200
4	2,750	8,000	18,150	67,750	125,950	6,050	13,850	47,550	103,850
5	2,900	8,100	19,350	71,150	127,800	6,100	14,900	52,700	104,350
6	2,950	8,400	20,600	71,550	145,800	6,100	16,700	58,350	110,850
7	2,950	8,850	21,100	71,950	155,200	6,150	19,100	60,000	117,900
8	3,050	8,950	21,200	73,100	172,300	2,200	20,200	60,000	120,550
9	3,050	9,100	21,200	74,700	172,550	6,900	20,200	61,800	129,250
10	3,200	9,250	22,550	74,700	172,650	7,800	20,450	62,250	131,950
11	3,200	9,300	22,900	75,650	177,500	7,800	21,000	65,850	135,950
12	3,250	9,350	23,650	78,850	178,200	8,600	23,100	67,500	137,350
13	(e)	9,750	24,300	82,250	182,850		24,350	67,600	140,350
14		9,800	25,200	83,800	183,300		24,650	69,150	142,400
15		10,000	27,000	86,350	195,800		25,000	70,200	143,200
16		10,350	27,250	96,000	196,900		25,150	72,000	149,950
17		10,350	27,450	98,450	203,000		28,250	75,800	159,300
18		10,550	27,550	107,400	205,050		32,100	77,850	167,750

[a]Additional stress ratios of 3.0 and 0.3 are being investigated.
[b]Mean stresses rounded to the nearest 500 psi.
[c]Variability σ_{sa} is the standard deviation of the distribution about the mean.
[d]Coefficient of variation K_V is the ratio of the variability to the mean expressed as a percentage
[e]Cycles-to-failure rounded to the nearest 50 cycles.

Table 3 - Tensile Strength Data for Notched Specimens[a]

($r_s = 0$)

Test No.	K_{ut}, klb	L_b, klb	S_{ut}, kpsi (b, c)	S_b, kpsi (b, c)
1	49.3	47.0	253.5	305.0
2	49.6	47.0	255.0	305.0
3	49.4	46.3	254.0	299.5
4	50.3	47.4	259.0	299.5
5	48.8	46.0	251.0	306.5
6	49.2	36.0	253.0	302.5
7	49.6	46.8	255.0	304.5
8	49.8	47.1	256.0	305.5
9	50.5	47.7	260.0	309.5
10	49.9	47.5	256.5	302.0

[a]Specimen diameter at the base of the notched was controlled to 0.4975 ±0.0025 in. which gives an area of 0.1944 ±0.0019 in.^{2}.

[b]Strength has been rounded to nearest 500 psi.

[c]Data reduction to normal parameters.

	Ultimate	Breaking
μ	255.5	304.0 kpsi
σ	2.5	3.0 kpsi

Table 4 - Tensile Strength Data for Unnotched Specimens

$r_s = 0$

Test No.	L_y, klb	L_{ut}, klb	L_b, klb	d, in. (a)	$\overline{A}$, in.^{2}	S_y, kpsi (b,c)	S_{ut}, kpsi (b,c)	S_b, kpsi (b,c)
1	31.5	32.5	24.3	0.4753	0.1774	177.5	183.0	264.0
2	30.6	31.5	23.5	0.4764	0.1784	171.5	176.5	254.0
3	30.6	31.6	23.3	0.4754	0.1775	172.5	178.0	249.0
4	20.8	31.0	22.8	0.4755	0.1776	168.0	174.5	251.5
5	29.9	31.1	22.9	0.4758	0.1778	168.0	175.0	254.5
6	30.1	31.3	23.2	0.4722	0.1752	172.0	178.5	256.5
7	30.4	32.5	25.0	0.4787	0.1800	169.0	181.0	256.0
8	30.2	31.4	24.2	0.4757	0.1777	170.0	178.0	253.0
9	30.6	31.6	23.6	0.4763	0.1782	172.0	177.5	259.0
10	30.3	31.2	24.2	0.4755	0.1766	171.0	176.5	250.5

[a]Specimen diameter was controlled to d ±0.0025 in. which gives an area of $\overline{A}$ ±0.0019 in.^{2}.

[b]Strengths rounded to the nearest 500 psi.

[c]Data reduction to normal parameters:

	Yield	Ultimate	Breaking
μ	171.0	178.0	255.0 kpsi
σ	3.0	2.5	4.5 kpsi

Table 5 - Endurance Strength Data

Material: SAE 4340 Steel, Rockwell 35/40
MIL-S-5000B, Condition C4

r_s	∞		3.0	1.03		0.3	0
n	W_p, lb (b)	R b = 1 c' = 7 (a)		W_p, lb. (b)	R b = 4 c' = 6 (a)		
1	25	S	Data Needed	26	S	Data Needed	Tensile testing no data required
2	26	F		27	F		
3	25	S		26	S		
4	26	F		27	F		
5	25	F		26	F		
6	24	S		25	S		
7	25	F		26	S		
8	24	S		27	F		
9	25	S		26	F		
10	26	F		25	F		
11	25	S		24	F		
12	26	F		23	S		
13	25	S		24	S		
14	26	F		25	S		
15	25	S		26	F		
16	26	S		25	S		
17	27	S		26	S		
18	28	F		27	S		
19	27	F		28	F		
20	26	S		27	F		
21	27	F		26	S		
22	26	F		27	F		
23	25	S		26	F		
24	26	F		25	S		
25	25	S		26	F		
26	26	F		25	S		
27	25	S		26	F		
28	26	F		25	F		
29	25	F		24	S		
30	24	F		25	S		
31	23	S		26	S		
32	24	F		27	S		
33				28	S		
34				29	F		
35				28	F		
36				27	F		
37				26	S		

[a] Test results:

S - success, if specimen survived $b\times10^{c'}$ cycles

F - failure, if specimen broke before $b\times10^{c}$ cycles

[b] Data reduction to normal parameters:

r_s	∞	3.0	1.03	0.3	0	
μ	56.0		56.5		Tensile	kpsi
σ	3.5		3.2		Test	kpsi

Table 6 - Strength Distribution Best Fit Summary

Material: SAE 4340 Steel, Rockwell 35/40
MIL-S-5000B, Condition C4

r_S	s_a kpsi (a)	Eqs. (b)		r' (c)	
		Normal	Lognormal	Normal	Lognormal
∞	144.0	Y=490X+2780	Y=6.39X+8.14	0.9022	0.8724
	114.0	Y=1070X+9030	Y=7.02X+9.09	0.9869	0.9821
	98.0	Y=3950X+22,170	Y=8.37X+10.03	0.9797	0.9772
	81.5	Y=12,750X+77,980	Y=9.50X+11.28	0.9612	0.9786
	73.0	Y=34,900X+161,980	Y=10.65X+12.00	0.9631	0.9426
3.0	Values Needed				
0.825	111.0	Y=1000X+6550	Y=6.95X+8.77	0.9291	0.9435
	91.7	Y=5780X+20,470	Y=8.79X+9.92	0.9779	0.9752
	75.5	Y=11,550X+61,030	Y=9.52X+11.03	0.9818	0.9665
	65.0	Y=21,930X+127,580	Y=10.1X+11.77	0.9777	0.9763
0.3	Values Needed				
0	0	Y=255,360X+2830	Y=12.48X+7.97	0.9891	0.9894

[a] Stress rounded to nearest 500 psi.

[b] Equation of a straight line on special graph paper with ordinate (Y), abscissa (X), slope (m) and intercept (b) such that Y=mX+b.

[c] The coefficient of correlation between two perfectly correlated linear variables should be 1.0000. Deviation from this value is a measure of goodness of fit.

Table 7 - Alternator Inner Rotor
Shaft Design Summary

Parameter	Conventional	Probabilistic
k_a	0.65	N.R.[b]
k_b	0.85	N.R.
k_c	0.752	D.M.[b]
k_d	0.867	(0.894; 0.009)[a]
k_e	0.680 min	N.R.
S_e'	9.35×10^4 psi	$(5.65\times10^4;\ 3.2\times10^3)$ psi
S_e	2.28×10^4 psi	$(5.08\times10^4;\ 2.75\times10^3)$ psi
T	1000 lb-in.	$(1\times10^3;\ 80)$ lb-in.
M	126 in.-lb	$(1.26\times10^2;\ 11.4)$ in.-lb
s_a	$\frac{1.28\times10^3}{\bar{d}^3}$ psi	$\frac{1.28\times10^3}{\bar{d}^3};\ \frac{116.4}{\bar{d}^3}$ psi
s_m	$\frac{8.83\times10^3}{\bar{d}^3}$ psi	$\frac{8.83\times10^3}{\bar{d}^3};\ \frac{709}{\bar{d}^3}$ psi
S_a	12.3×10^3 psi	$(1.91\times10^4;\ 1.10\times10^3)$ psi
S_m	85.0×10^3 psi	$(1.31\times10^5;\ 1.21\times10^3)$ psi
d	19/32 in.	15/32 in.
R	(c)	0.999

[a] Normal distribution notation (mean, standard deviation).

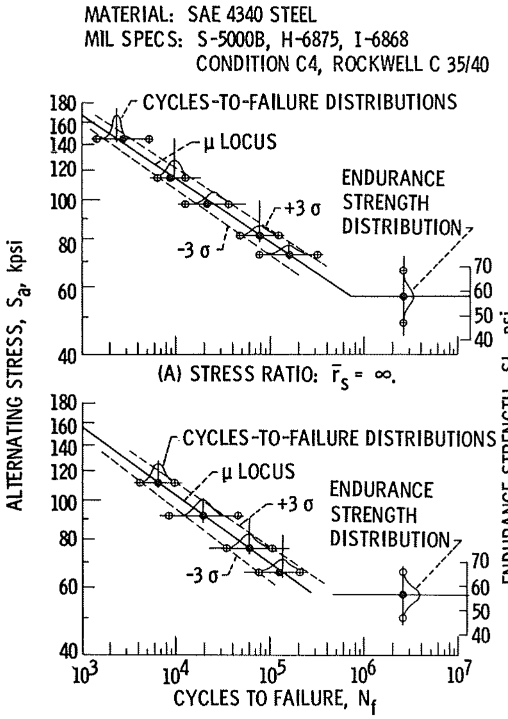
[b] Legend:

N.R. - not required as experimental data includes this factor.

D.M. - a different method is used to include this factor.

[c] The reliability obtained by the approach of Shigley (14, p. 169) is not theoretically applicable to this complex case, hence the reliability is really not known.

E-5528



(B) STRESS RATIO: $\bar{r}_s$ = 0.825 FOR CYCLES-TO-FAILURE AND $\bar{r}_s$ = 1.03 FOR ENDURANCE.

Figure 1. - Distributional S-N diagrams.

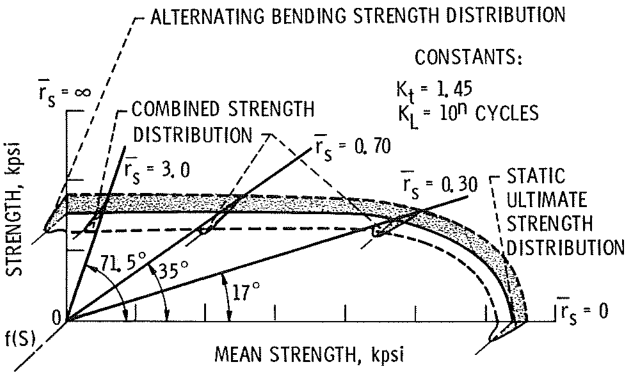
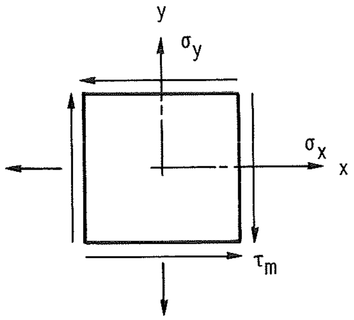


Figure 2. - Three-dimensional Goodman fatigue diagram.

E-5528



EQUATIONS

$$\sigma_x = \sigma_{xm} \pm \sigma_{xa}$$
$$\sigma_y = \sigma_{ym} \pm \sigma_{ya}$$
$$\tau_m = \tau_{xym} \pm \tau_{xya}$$

Figure 3. - Rotating shaft surface element stresses.

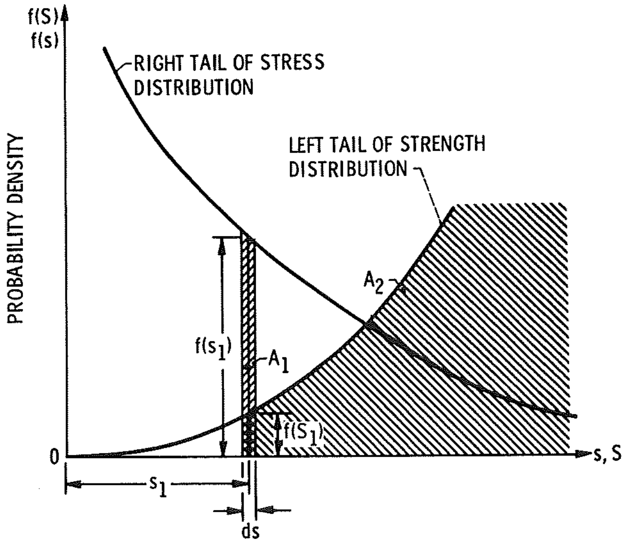


Figure 4. - Probability areas of stress and strength.

E-5528

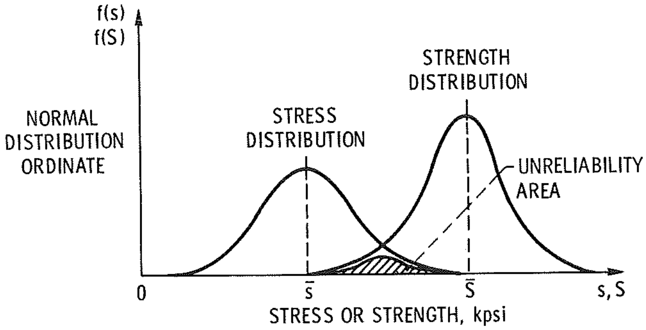


Figure 5. - Stress and strength distributions.

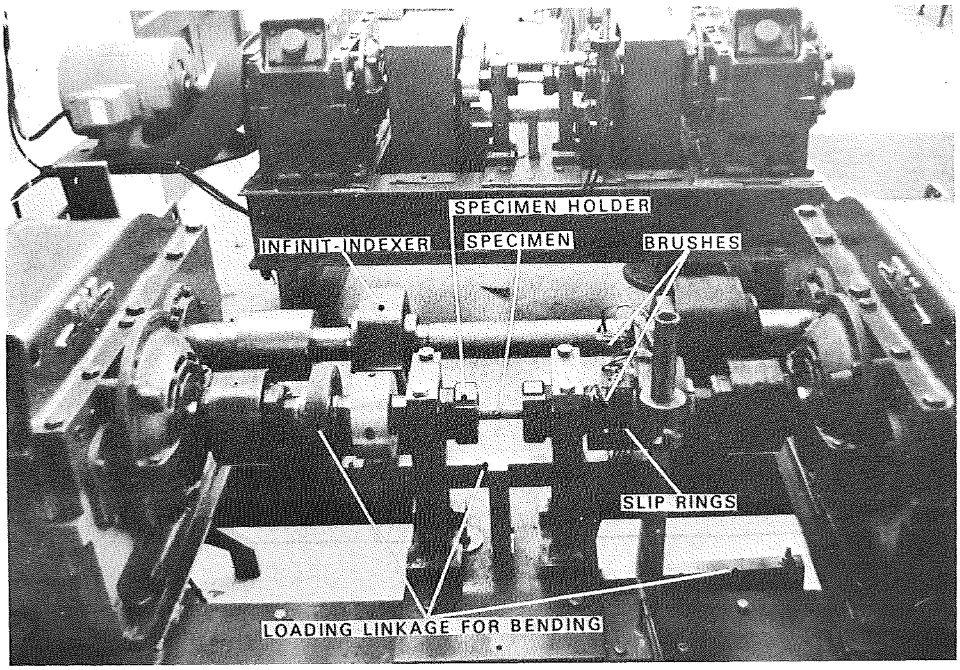
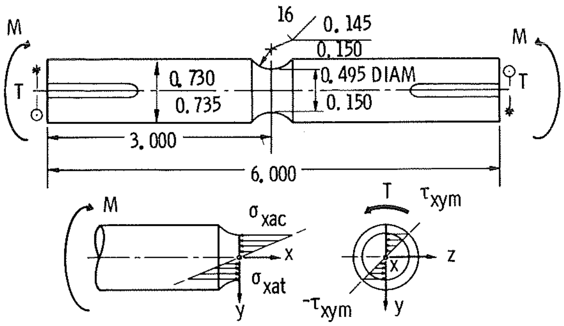
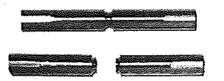


Figure 6. - A close-up view of the complex-fatigue research machines.

E-5528

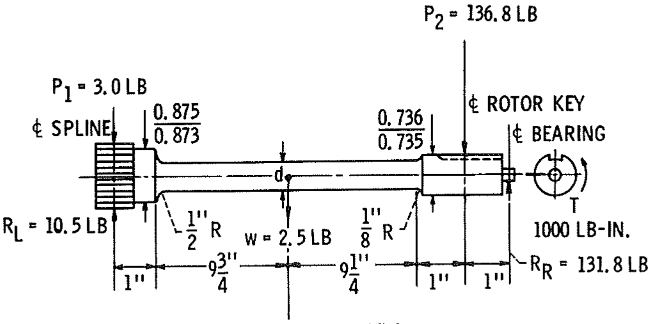


(A) DIMENSIONS, FINISH, FORCES AND STRESSES.

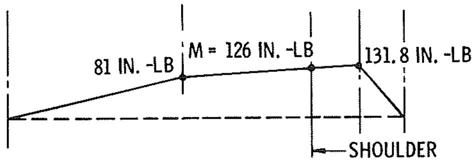


(B) CONFIGURATION BEFORE AND AFTER FATIGUE TESTING.

Figure 7. - Test specimen details.



(A) INNER SHAFT DETAILS.



(B) BENDING MOMENT DIAGRAM.

Figure 8. - Alternator inner rotor shaft.

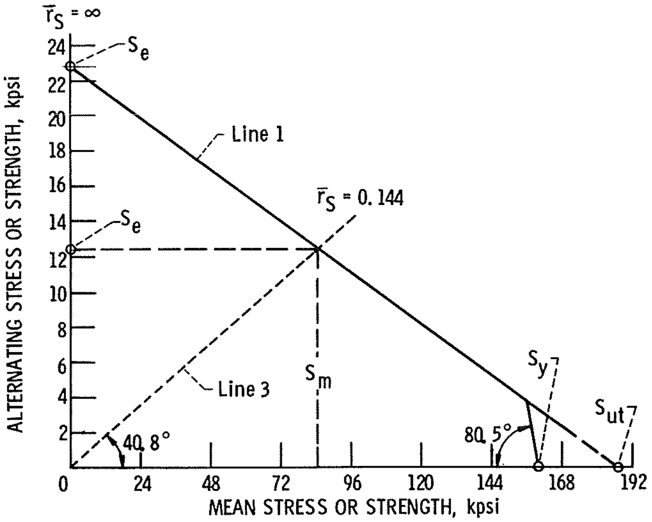


Figure 9. - Conventional modified Goodman diagram.

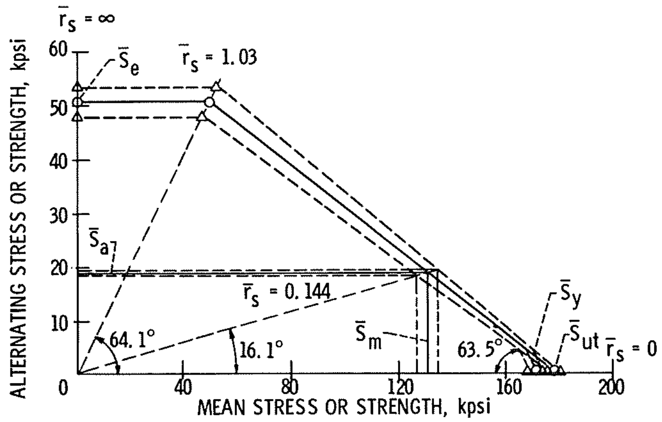


Figure 10. - Probabilistic modified Goodman diagram.